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SOME NEW ASPECTS OF THE APPLICATION OF THE ADJOINT FUNCTIONS AND OF THE PERTURBATION THEORY IN REACTOR AND SHIELDING DESIGN.

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The application of perturbation theory formulas for a critical nuclear reactor, published e.g. in Refs /1/, /2/, /3/, /4/, /5/, proved to be rather suitable in reactor design and experimental research.

Refs /6/, /7/ extended the area of perturbation theory application for arbitrary linear functionals in stationary and time depending problems of the radiation transfer with given sources. In Ref /8/ the perturbation theory formula is obtained for a ratio of different processes numbers in a stationary reactor.

The application of formulas from Ref /8/ giving more possibilities in an investigation of reactor physical characteristics is discussed in chapter I (L.N.Usachev, S.M.Zaritsky).

The example of this theory applying for breeding ratio calculations for variants of fast breeder is presented. The possibility of using this theory for improvement of the multigroup constants system on the basis of results measured in a critical assembly is considered.

The use of perturbation theory formulas for the effective multiplication constant and for ratio of different processes numbers in a reactor for the statement of variational problems on optimum distribution of different materials in a reactor with an arbitrary neutron spectrum is discussed in chapter II. (V.Ya.Pupko, G.I.Drujinina). Some identical problems for K_{eff} in applying to a thermal reactor were considered in Ref/4/.

Chapter III (V.V.Orlov, A.A.Abagian, R.P.Fedorenko, A.A.Dubinin, A.P.Suvorov) is devoted to using of perturbation theory formulas for the shielding characteristics optimization.

As the application of classical calculus of variations methods for shielding problems proved to be difficult in some cases, the optimization process is made on the base of linear programming ideas.

C h a p t e r I. THE PERTURBATION THEORY FOR A RATIO OF PROCESSES NUMBERS IN A REACTOR.

§1. Summary and some improvements in the derivation made in Ref /8/.

The well known in the reactor theory perturbation theory relation for determination of disturbed critical reactor parameters (see for instance /3/)

$$-\int\!\!\int\Big[\frac{1}{\ell'_{aifs}(\vec{r},E)}-\frac{1}{\ell_{aifs}(\vec{r},E)}\Big]F'(\vec{r},E,\vec{n})F_x^+(\vec{r},E,\vec{n})\,d\Omega\,dE\,dV+\int\!\!\int\big[w'(E,E',\vec{n}\vec{n}',\vec{r})-$$

$$-w'(E,E',\vec{n}\vec{n}',\vec{r})\big]F'(\vec{r},E,\vec{n})F_x^+(\vec{r},E,\vec{n})\,d\Theta+\int\!\!\int\Big[\frac{\nu'(\vec{r},E',\vec{n}')}{\zeta'_f(\vec{r},E')}\chi'(E,E',\vec{n}\vec{n}',\vec{r})-\frac{\nu(\vec{r},E',\vec{n}')}{\zeta_f(\vec{r},E')}\chi(E,E',\vec{n}\vec{n}',\vec{r})\Big]\times$$

$$\times F'(\vec{r},E',\vec{n}')F_x^+(\vec{r},E,\vec{n})\,d\Theta=0\,,\qquad \text{where } d\Theta=dV\,dE\,dE'\,d\Omega\,d\Omega' \tag{1}$$

is obtained from the equation for neutron flux $F'(\vec{r},E,\vec{n})$ in a disturbed reactor with primed parameters:

$$\frac{\partial F'(\vec{r},E,\vec{n})}{\partial s}+\frac{F'(\vec{r},E,\vec{n})}{\ell'_{aifs}(\vec{r},E)}-\int w'(\vec{r},E,E',\vec{n}\vec{n}')F'(\vec{r},E',\vec{n}')\,dE'd\Omega'=\int\frac{\chi'(\vec{r},E',\vec{n}')}{\zeta'_f(\vec{r},E')}\nu(\vec{r},E')\chi(E,E',\vec{n}\vec{n}',\vec{r})\,dE'd\Omega' \tag{2}$$

and from the equation for neutron importance in an undisturbed reactor

$$-\frac{\partial F_x^+(\vec{r},E,\vec{n})}{\partial s}+\frac{F_x^+(\vec{r},E,\vec{n})}{\ell_{aifs}(\vec{r},E)}-\int w(\vec{r},E,E',\vec{n}\vec{n}')F_x^+(\vec{r},E',\vec{n}')\,dE'd\Omega'=\int\frac{F_x^+(\vec{r},E,\vec{n}')}{\zeta_f(\vec{r},E)}\nu(\vec{r},E)\chi(E,E',\vec{n}\vec{n}',\vec{r})\,dE'd\Omega' \tag{3}$$

To obtain the relation (1) it is necessary to cross multiply equations /2/ and /3/ by F^+ and F', subtract one from another and integrate over all energies, velocity directions and reactor volume up to the outer boundary with vacuum.

Identical operations with co-adjoint equations, formulated in Ref /6/, were used in Ref /7/ to obtain the perturbation theory relation for an arbitrary linear functional of neutron flux. There was considered the more general time depending problem on the distribution of neutrons from given sources.

The concept of the neutron importance in any previous n^{th} cycle to a process in a chosen (zero) cycle F_n^+ was introduced in Ref /8/ to obtain the perturbation theory relation for a ratio of processes numbers occurred in a stationary reactor which is described by homogeneous equations. The set of equations for these neutron importances serves as adjoint equation in derivation of the relation required. If we are interested, for example, in the neutron capture in i^{th} isotope this set of equations is

$$-\frac{\partial F_0^+}{\partial s}+\frac{F_0^+}{\ell_{aifs}}-\int wF_0^+\,dE'd\Omega'=\frac{1}{\ell_a^i(\vec{r},E)} \tag{4a}$$

$$-\frac{\partial F_1^+}{\partial s}+\frac{F_1^+}{\ell_{aifs}}-\int wF_1^+\,dE'd\Omega'=\frac{\nu}{\zeta_f}\int\chi F_0^+\,d\Omega'dE' \tag{4b}$$

$$-\frac{\partial F_n^+}{\partial s}+\frac{F_n^+}{\ell_{aifs}}-\int wF_n^+\,dE'd\Omega'=\frac{\nu}{\zeta_f}\int\chi F_{n-1}^+\,d\Omega'dE' \tag{4c}$$

The successive solving of equations (4a), (4b), (4c) is equivalent to a usual procedure of solving of equation (3) by the successive iterative method. Hence, it is clear, that at $n \to \infty$ $F_n^+ \to \lambda F_x^+$. Using the adjointness of left parts of equations (3), (4a), (4b), (4c) and left part of the equation for the undisturbed flux $F(\vec{r}, E, \vec{n})$ (equation /2/ being without primes) and carrying out the procedure of the cross multiplying and subtraction, as it was indicated above, we obtain the set of equations:

$$a_i \equiv \int \frac{F}{\ell_{ai}} d\Omega dE dV = \ldots = \int \frac{\nu\chi}{\ell_f} F_n^+ F_i d\theta = \ldots = \lambda_i \int \frac{\nu\chi}{\ell_f} F_x^+ F \, d\theta \quad (4d)$$

Hence $\lambda_i = \frac{a_i}{I_{fn}}$, where $I_{fn} = \int \frac{\nu\chi}{\ell_f} F_x^+ F \, d\theta$

Summarizing equations for F_0^+, F_1^+... to F_n^+, we obtain the equation for $\Phi_n^+ = \sum_\alpha^n F_\alpha^+$

$$-\frac{\partial \Phi_n^+}{\partial s} + \frac{\Phi_n^+}{\ell_{aifs}} - \int w \Phi_n^+ dE' d\Omega' - \int \frac{\nu\chi}{\ell_f} \Phi_n^+ dE' d\Omega' = -\int \frac{\nu\chi}{\ell_f} F_n^+ dE' d\Omega' + \frac{1}{\ell_i} \quad (5)$$

Carrying out the same procedure of the cross multiplying, integration with equations (2) and (5), then forming the change of the number of processes under consideration which is equal to

$$\delta a_i = a_i' - a_i = \int \frac{F'}{\ell_{ai}'} dE d\Omega dV - \int \frac{F}{\ell_{ai}} dE d\Omega dV$$

and using the relation (4d):

$$\int \frac{F}{\ell_{ai}'} dE d\Omega dV = \int \frac{\nu\chi}{\ell_f} F_n^+ F \, d\theta$$

we obtain

$$\delta a_i = \int F' \left(\frac{1}{\ell_{ai}'} - \frac{1}{\ell_{ai}} \right) dE d\Omega dV + \int F' \Phi_{in}^+ \left(\frac{1}{\ell_{aifs}'} - \frac{1}{\ell_{aifs}} \right) dE d\Omega dV - \int F' \Phi_{in}^+ (w' - w) d\theta -$$

$$- \int \left(\frac{\nu\chi'}{\ell_f'} - \frac{\nu\chi}{\ell_f} \right) F' \Phi_{in}^+ d\theta + \int \frac{\nu\chi}{\ell_f} F_n^+ (F' - F) d\theta \quad (6)$$

Considering only perturbations which remain reactor critical, i.e. perturbations satisfying the condition (1), we note that in second, third and fourth terms of the right part of (6) it is possible to replace $\Phi_n^+ = \sum_\alpha^n F_\alpha^+$ which tends to infinity at $n \to \infty$ by $\sum_\alpha^n \left(F_\alpha^+ - \frac{a_i}{I_{fn}} F_x^+ \right)$, which has the definite limit

$$\Phi^+ = \sum_\alpha^\infty \left(F_\alpha^+ - \frac{a_i}{I_{fn}} F_x^+ \right) \quad (7)$$

The last term in the right part is equal to

$$\frac{a_i}{I_{fn}} \int \frac{\nu\chi}{\ell_f} F_x^+ (F' - F) d\theta$$

Only ratios of processes numbers (for instance the ratio of capture-to-fission numbers, i.e. to the reactor power) as well as changes of these ratios when making perturbations in reactor have the physical meaning in a critical reactor.

The variation of ratio of capture number in isotope-i to this in isotope-j which equals

$$\delta \left(\frac{a_i}{a_j} \right) = \frac{a_i + \delta a_i}{a_j + \delta a_j} - \frac{a_i}{a_j} = \frac{a_i}{a_j} \left(\frac{\delta a_i}{a_i} - \frac{\delta a_j}{a_j} \right) \frac{a_j}{a_j'} \quad (8)$$

is possible to write using (6) at $n \to \infty$ and (7) in the following form

$$\delta\left(\frac{a_i}{a_j}\right) = \frac{a_i}{a_j}\left\{\iiint\left[\frac{F''}{a_i}\left(\frac{1}{\ell_{ai}}-\frac{1}{\ell_a}\right)-\frac{F'}{a_j}\left(\frac{1}{\ell_{aj}}-\frac{1}{\ell_a}\right)\right]dE\,d\Omega\,dV+\int F''\psi^+\left(\frac{1}{\ell_{ajs}}-\frac{1}{\ell_{ajs}}\right)dE\,d\Omega\,dV-\right.$$

$$\left.-\int F''\psi^+(w'-w)\,d\theta-\int\left(\frac{\nu'\chi'}{\ell_f}-\frac{\nu\chi}{\ell_f}\right)F''\psi^+\,d\theta\right\} \qquad (9)$$

Here

$$F'' = F'\frac{a_j}{a_j'} \qquad (10)$$

is the neutron flux in a disturbed reactor normalized in such a way that capture number in isotope-j is equal to this number in an undisturbed reactor, and

$$\psi^+ = \frac{\phi_i^+}{a_i}-\frac{\phi_j^+}{a_j} \qquad (11)$$

is a function having the meaning of neutron importance with respect to ratio of processes numbers as it is seen from relation (9).

From relation (11) and the linearity of set of equations (4) it is clear that substituting the linear combination $\left(\frac{1}{a_i\ell_i}-\frac{1}{a_j\ell_j}\right)$ instead of $1/\ell_a^i$ in the right part of equation (4a) and successively solving equations (4) we shall directly obtain ψ^+ as a sum $\sum_\alpha^3 E_\alpha^+$. Only one set of equations is solved instead of two and there is no need for subtracting the main harmonic F_x^+ from each F^+ as it is done in relation (7), since it is generated by every item in the right part of the equation with the same coefficient $1/I_{fn}$ but with a different sign. This combination which is advisable to use in calculations was pointed out in Ref 13. The equation for the neutron importance with respect to linear-fractional functional was derived in this reference based upon the introduced there neutron importance concept with respect to an arbitrary non-linear functional and upon the neutron importance balance which had been used in Ref 3 and 7. For small disturbances it was shown that the functional derivative of the mentioned non-linear functional with respect to neutron flux should be in the right hand of the equation for the neutron importance. In the case of the linear-fractional functional, under consideration, this derivative is equal to

The algorithm of the perturbation theory described was put in practice by us using 18-group diffusion approximation in the program for digital computer. All examples of the perturbation theory application described below have been realized or are being realized now by means of this program.

§2. The examples of the use of perturbation theory formulas.

A. Reactor variant calculations. Reactor design variations are usually described, from the neutron physics standpoint, by change of the material concentration in reactor different points. Hence, for reactor variant calculations by means of formulas (1) and (2) reactor parameters

variations are presented as: $\frac{1}{\ell_{aijs}} - \frac{1}{\ell'_{aijs}} = \sum_\alpha \delta\rho_\alpha \sigma^{(\alpha)}_{aijs}$; $w' \cdot w = \sum_\alpha \delta\rho_\alpha w^{(\alpha)}$;

$\frac{\nu' \chi'}{\ell'_f} - \frac{\nu\chi}{\ell_f} = \sum_\alpha \delta\rho_\alpha \left(\frac{\nu\chi}{\ell_f}\right)^{(\alpha)}$

where $\sigma^{(\alpha)}_{aijs}, w^{(\alpha)}, \left(\frac{\nu\chi}{\ell_f}\right)^{(\alpha)}$ define the interaction between neutrons and α-type nucleus.

To show that perturbation theory formulas are rather suitable in the applying for variant calculations we shall write them for a given reactor. Let us consider a fast reactor which is the spherical simplified model of a power breeder. In the core with a radius 66 cm and with a volume 1204 litres fuel, i.e. uranium and plutonium oxide, occupies 35% of all volume, steel - 25% and sodium - 40%. Nuclear concentrations of oxygen, plutonium, uranium, steel and sodium (multiplied by 10^{24}) in 1 cm^3 are correspondingly equal to $\rho_o^I = 0.01288$, $\rho_g^I = 0.001108$, $\rho_\delta^I = 0.005334$, $\rho_{ss}^I = 0.0214$, $\rho_{Na}^I = 0.00888$.

In the breeding blanket with a thickness 60 cm concentrations are: $\rho_o^{II} = 0.0272$, $\rho_g^{II} = 0$, $\rho_\delta^{II} = 0.01355$, $\rho_{ss}^{II} = 0.0153$, $\rho_{Na}^{II} = 0.00483$. In the steel reflector with a thickness 20 cm. $\rho_o^{III} = 0$, $\rho_g^{III} = 0$, $\rho_\delta^{III} = 0$, $\rho_{ss}^{III} = 0.072$, $\rho_{Na}^{III} = 0.00345$. The total breeding ratio BR and the breeding of the core BRC are correspondingly equal to 1.4575 and 0.5947.

At uniform variations of different materials concentrations in the core and in the breeding blanket it is suitable to present relations describing variations of the plutonium critical concentration, BR and BRC in the next form:

$$\delta\rho_g^I = \sum_N \sum_{\alpha \neq g} L^{\alpha N} \delta\rho_\alpha^N + L_\delta^I (\delta\rho_\delta^I + \delta\rho_g^I); \quad \frac{\delta BR}{BR} = \sum_N \sum_\alpha L_{BR}^{\alpha N} \delta\rho_\alpha^N; \quad \frac{\delta BRC}{BRC} = \sum_N \sum_\alpha L_{BRC}^{\alpha N} \delta\rho_\alpha^N \quad (12),$$

here N is the reactor zone number. Coefficients L, L_{BR}, L_{BRC} of formulas (12) are presented in the table:

L \ α	Pu^{239}	U^{238}	Na	Steel	O	C	Fission fragments	Mo	Ni
L^I	-	0.0411	-0.0^2132	-0.0^3441	-0.0^2194	-0.0^2191	0.0689	0.0204	-0.0^2168
L^{II}	-0.119	0.0^3156	-0.0^2104	-0.0^2137	-0.0^3795	-0.0^3480	-0.0^3674	-0.0^2105	-0.0^2223
L^I_{BR}	-884	68.1	-9.44	-8.15	-8.73	-10.6	-7.95	-8.84	-11.5
L^{II}_{BR}	303	5.00	-1.76	-3.25	-0.948	-0.562	-53.7	-17.6	-5.44
L^I_{BRC}	-978	182.4	5.45	2.27	5.22	6.95	-0.767	1.05	1.60
L^{II}_{BRC}	5.99	-0.363	0.284	0.214	0.255	0.281	0.382	0.163	0.422

These coefficients were calculated for small disturbances, i.e. in assumption F'=F and F''=F. The mentioned programme permits to make the calculation with F' and F'' to determine limits of these formulas application and to clear up causes of possible disagreements.

B. Effect of the constants inaccuracy. At the calculations of coefficients for relations (12) we obtain also coefficients defining the effect of any material constant inaccuracy in any group on the critical concentration, total breeding ratio and core breeding ratio. It is possible to perform the analysis similar to that performed in Ref /14/ by means of these coefficients. It will be recalled that mentioned coefficients in Ref /14/ were determined by the direct calculation.

C. Analysis of critical assembly experiments for the constant system improvement. Perturbation theory equations (1) and (9) may be considered as linear equations, relating differences of observed and calculated values with inaccuracy in the multigroup constant system, which, as it is proposed, causes the calculation and the experiment discrepancy. The application of the perturbation theory relations with above meaning for analysis of critical assembly experiments and for such an experiment planning can give the systematic improvement method of the multigroup constant system for a reactor design. For each critical assembly on which experiments are performed it is possible to calculate the linear equation coefficients, relating the critical concentration variation and the changes of ratios of different processes numbers observed in an experiment to multigroup constant variations. Having a large number of the mentioned relations for several critical assemblies and for many ratios of processes numbers it is possible to state the matching problem of constants, describing all experiments in the best way. In the general statement such problems can be solved by the linear programming methods /11/ and in particular simple cases they can be solved by the trial and error method.

Capture-to-fission number ratios in different isotopes measured in different points and integrally over reactor zones can be used as ratios of processes numbers. Such easily measured values as the reactivity at the placing of different samples can be used too. Indeed, the reactivity caused by a sample placed in a critical reactor can be written so (see /3/)

$$\frac{K-1}{K} = \frac{a_i}{a_j} \quad , \text{ where } a_i \equiv -\int \frac{FF_x^+}{\rho_{samp}} d\Omega\, dE\, dV + \int_{V_{samp}} FF_x^+ d\theta + \left(\frac{\nu\chi}{\ell_f}\right)_{samp} FF_x^+ d\theta \qquad (13)$$

is an additional neutron importance arisen per unit of time at the sample placing,

$$a_j \equiv I_{fn} \equiv \int \left[\frac{\nu\chi}{\ell_f} + \left(\frac{\nu\chi}{\ell_f}\right)_{samp} \right] FF_x^+ d\theta \qquad (14)$$

is the importance of fission neutrons emissioned throughout a reactor per unit of time. Hence, it is clear that to calculate reactivity variations $\delta\left(\frac{k-1}{K}\right)$ it is possible to use the same algorithm as one

for other ratios of processes numbers. It is necessary to note that in this case values which at multiplying by neutron flux F and integrating over energies, velocity directions and reactor volume will give a_i and a_j, determined from equations (13) and (14), should be in right hand of equation (4a) instead of $1/la_i$ and $1/la_j$. Therefore, it is necessary to replace in equation (4a) and in following equations:

$$\frac{1}{\ell_a^i} \longrightarrow -\frac{F_x^+}{\ell_{a\,i\,fs}^{samp}} + \int w_{samp}(E',E,\vec{n}\vec{n}',\vec{r})F_x^+(\vec{r},E',\vec{n}')dEd\Omega' + \iint\left(\frac{\nu\chi}{\ell_f}\right)_{samp} F_x^+ dE'd\Omega' \qquad (15)$$

$$\frac{1}{\ell_a^i} \longrightarrow \int\left[\frac{\nu\chi}{\ell_f} + \left(\frac{\nu\chi}{\ell_f}\right)_{samp}\right]F_x^+ dE'd\Omega' \qquad (16)$$

Formulas (9), (15) and (16) take into account not only possible inaccuracy in sample constants but inaccuracies of the constants describing the reactor and also the reactor spectrum variations caused by the influence of the inserted sample.

The variation of the prompt neutron mean life time being a ratio of the total reactor neutron importance to the importance of fission neutrons, emitted per second, can be obtained identically /3/. In this case in formulas (4) - (9) it is necessary to replace.

$$a_i \longrightarrow \int \frac{FF_x^+}{V} dE\,d\Omega\,dv \;;\quad 1/\ell_a^i \longrightarrow \frac{F_x^+}{V}$$

$$a_j \longrightarrow \int F F_x^+ \frac{\nu\chi}{\ell_f} d\theta \;;\quad 1/\ell_a^j \longrightarrow \int F_x^+ \frac{\nu\chi}{\ell_f} dE\,d\Omega$$

Such analysis was already begun to carry out at the treatment of experimental data obtained in assemblies /15./

The development of the perturbation theory described considerably extends its application area which of course is not limited by presented examples (See e.g. /8/). In particular, it becomes possible to set the problem of optimization of breeding ratio or some other ratios of processes numbers which is discussed in the next chapter.

Chapter II. THE USING OF THE PERTURBATION THEORY FUNCTIONALS IN THE VARIATIONAL PROBLEMS ON MATERIAL OPTIMUM DISTRIBUTION IN NUCLEAR REACTORS.

Using the perturbation theory it is possible to state in general case the problem on a different reactor materials optimum distribution with relation to the achievement of most advantageous characteristics caused by the effect of these materials. The problem on reactor critical mass minimization and the problem on the maximum breeding ratio in a breeder are examples of such problems. The approach to the variational problem consists in the finding of extremum of corresponding funotional which has variation presented with perturbation theory formulas. Several other functionals or nonintegral conditions can be simultaneously fixed so as from calculus of variations point of view we shall deal with the isoperimetrical or conditional extremum problem /16/. In the

first from mentioned problems the functional is the reactor effective multiplication constant K_{eff} having the variation being written with the perturbation theory formulas /3/. In the second problem perturbation theory formulas are used /3/ for the linear fractional functional which describes a ratio of two different reactor processes. Perturbation theory formulas contain different materials efficiency functions describing the effect of these materials content variation on a value of varying functional. This permits to formulate the variation problem in the optimum distribution investigation of different materials in a reactor.

The minimization reactor charge problem and more briefly the maximum breeding ratio problem are considered below. Similar to considered problems it is possible to formulate other variation problems on the optimization of reactor characteristics (the maximum compensating capacity of the absorption material, etc.).

§1. The statement of the problem on minimum critical reactor mass.

We have the nuclear reactor with limited sizes and with the given content of structural materials. At some uniform concentration of U-235 in the core the reactor has definite value of K_{eff} and critical mass G_5. How should the distribution of U-235 over all core be modified to obtain the critical mass reducing without K_{eff} variations, if it is possible?

Similar problem has been considered by Goetzel /17/ to the thermal reactor case with the same moderator in the core as in the reflector. The method of such reactor calculation is considered also in monographs /13/ and /4/.

In this report the condition of the critical mass minimization is considered in general case for a reactor with an arbitrary neutron spectrum.

To definition we assume that varying the concentration of U-235 $\rho_5(\vec{r})$ in the point $\vec{r}$ we simultaneously change the content of only one component "K" having the concentration $\rho_k(\vec{r})$. In general we have the possibility in principle to lower the critical mass of the given reactor from $\rho_5 = \text{const} < \rho_{5,0}$ with the corresponding redistribution of U-235 in the core. The problem is considered in two stages: at first the variational problem for maximum K_{eff} at the given reactor mass is stated and then the transition to problem on minimum G_5 at fixed K_{eff} is discussed.

§2. The achievement of maximum K_{eff} in a reactor with the given charge.

Varying $\rho_5(\vec{r})$ we shall find the maximum K_{eff} at the following iso-

perimetrical condition:

$$G_5 = C\int \rho_5(\vec{r})\, dV \qquad (1)$$

According to Euler's method /16/ we construct the functional

$$J(\rho_5) = K_{eff} + \lambda_1 G_5 \qquad (2)$$

and find its absolute extremum. The expression for the variation is written so:

$$\delta J(\rho_5) = \delta K_{eff}(\rho_5) + \lambda_1 \int_{V_0} \delta \rho_5(\vec{r})\, dV \qquad (3)$$

The first term we shall express with perturbation theory formulas

$$\delta K_{eff}(\rho_5) = \int_{V_0} f(\vec{r};\rho_5)\, \delta\rho_5(\vec{r})\, dV \qquad (4)$$

By symbol $f(\vec{r};\rho_5)$ it is defined the net neutron importance generated per second in the point $\vec{r}$ with one fissionable material nucleus as a result of all interaction processes of neutrons with U-235. The function $f(\vec{r};\rho_5)$ in the following text is named as efficiency function of U-235. Dependence of this function upon $\rho_5(\vec{r})$ is functional, its structure is:

$$f(\vec{r};\rho_5) = f^{(5)}(\vec{r};\rho_5) - \frac{\rho_{k,0}}{\rho_{5,0}} f^{(k)}(\vec{r};\rho_5) \qquad (5)$$

where $\rho_{5.0}$; $\rho_{k,0}$ - material concentrations corresponding to its maximum density. In general:

$$f^{(5)}(\vec{r};\rho_5) = \frac{K_{eff}}{J_{fn}}\left\{\int_{4\pi}\int_{4\pi}\int_{V_0} F(\vec{r},E,\vec{\Omega})\left[\sigma_f(E')\frac{\nu_f(E')}{K_{eff}}\chi(E,E',\vec{\Omega}\vec{\Omega}') + W^{(5)}(E'\to E,\vec{\Omega}\vec{\Omega}')\right] F^{+}(\vec{r},E,\vec{\Omega})\, dE'd\vec{\Omega}'dE\,d\vec{\Omega} - \int_{4\pi} F(\vec{r},E,\vec{\Omega}) F^{+}(\vec{r},E,\vec{\Omega})\sigma_{a,fs}^{(5)}(E)\, dE\, d\vec{\Omega}\right\}$$

$$f^{(k)}(\vec{r};\rho_5) = \frac{K_{eff}}{J_{fn}}\left\{\int_{4\pi}\int_{4\pi}\int_{V_0} F(\vec{r},E,\vec{\Omega}) W^{(k)}(E'\to E,\vec{\Omega}\vec{\Omega}') F^{+}(\vec{r},E,\vec{\Omega})\, dE'd\vec{\Omega}'dE\,d\vec{\Omega} - \int_{4\pi} F(\vec{r},E,\vec{\Omega})\,\sigma_{als}^{(k)}\, F^{+}(\vec{r},E,\vec{\Omega})\, dE\, d\vec{\Omega}\right\} \qquad (6)$$

where I_{fn} is the normalization importance of fission neutrons. Euler's equation for the variational problem under consideration

$$f(\vec{r};\rho_5) = const \qquad (7)$$

is received by using equations (3) and (4) from extremum condition

$$\delta J(\rho_5) = 0$$

Equation (7) is the necessary condition of an optimal distribution of the given amount of U-235 in the core, ensuring the maximum K_{eff} in a reactor with an arbitrary neutron spectrum*. The physical meaning of this condition is clear: if the U-235 efficiency function is constant throughout the core, every gram of the fissionable material in any point is used equivalently. From this it follows that here we consider such reactors, for which an extremum of functional $K_{eff}(\rho_5)$ exists at the concentration of U-235 $0 < \rho_5(\vec{r}) < \rho_{5.0}$.

Equation (7) permits to base the method of experimental determina-

* The same requirement for the minimization of the critical mass of a thermal reactor is noted in Ref /4/ by A.Weinberg and E.Wigner.

tion of the U-235 concentration distribution in reactor for which maximum K_{eff} is realized at the given charge. This equation can serve too as a proper solution criterion and should be used in numerical calculations for such reactors. These are accomplished with an iterative method, achieving on procedure reduction to constant value of efficiency function of U-235 in a core volume.

The possible calculating iterative procedure consists in gradual redistribution of U-235 in core in accordance with the shape of the calculated efficiency function of U-235.

§3. The transition from maximum K_{eff} problem to minimum charge problem.

Let functions $\rho_5^I(\bar{r})$, $\rho_5^{II}(\bar{r})$,... providing maximum K_{eff}^I, K_{eff}^{II} have been found for different charge values G_5^I, G_5^{II}... of the reactor under consideration. From Fig.1 it follows the reactor charge corresponding to curve 1 with $(\partial K_{eff}/\partial G_5)_{Va.3} > 0$ for any value of K_{eff} required will be the least. In connection with this it is necessary specially to analyse the sign of derivative $(\partial K_{eff}/\partial G_5)_{Va.3}$ in the different reactors.

Using the perturbation theory formulas it can be shown that:

$$\left(\frac{\partial K_{eff}}{\partial G_5}\right)_{Va.3} = \frac{const}{V_{a.3}} \int_{V_{a.3}} f\, dV = const\, \bar{f}$$

The critical reactor mass minimization at $\bar{f} < 0$ requires an additional consideration. Identically to a proof in Ref /10/ for reactors with ρ_5=const it can be shown that for the different types of reactors with variable concentration of U-235 the details of function diagram G_5=f(Va.3) are determined with a relation the neutron leakage factor from a reactor and an efficiency fissionable material factor $\eta \sim \bar{f}$ in a reactor. Schematical diagrams of dependence G_5=f(Va.3) for the different spectrum reactors are given in Fig.2. In all cases but the part CD of the curve the U-235 efficiency factor $\eta > 0$. In the part CD a depreciation of U-235($\eta < 0$) occurs and the slope of the curve G_5=f(Va.3) varies. Causes of such event can be, for instance, a higher radiative capture of neutrons in the fissionable material, an availability of the strong resonance absorbers in reactor, the using of a hydrogenous moderator, etc. The curve slope change of G_5=f(Va.3) in part CD leads to that without varying the core volume certain amount of U-235 can be removed from such reactors replacing it by a moderator without reactivity loss. Thus the critical reactor mass minimization at $\eta < 0$ is reduced pro forma to transition from part CD to part C'D of the diagram curve.

§4. On fission density non-uniformity in minimized charge reactors.

For minimizing a charge it serves to purpose to displace U-235 from peripheral reactor regions near to central regions, however such fission material redistributing in reactor is able adversely to effect on the non-uniformity coefficient of heat generation in a core. At critical mass minimizing in a small reactor it is necessary to make a "local" rising of U-235 concentration on periphery of a core (Ref 17) parallel with the fissionable material redistributing to reactor center and some improvement of fission density non-uniformity takes place.

In case a fission density maximum is at the boundary of the reflector and the core due to a reflector moderating effect U-235 displacement from peripheral regions of the core to central regions results in both improvement of the heat generation non-uniformity and the reactor charge gain.

In a thermal heterogeneous reactor radial flattening of the average energy generation in separate fuel elements may be reached by gradual moderator volume per cent rising from centre to the periphery of the core. Parallel with this it is possible to uniform U-235 efficiency function through fuel elements. As the energy release non-uniformity problem is of importance in engineering, the variational problem of critical reactor mass minimization with the fission density non-uniformity coefficient limiting in the core may be formulated.

The derivation on Euler's equation for this problem is identical to one mentioned above. In deriving this equation one takes into account an isoperimetric condition for the volume non-uniformity coefficient K_v, while an expression for $\delta K_v(\rho_5)$ may be obtained on the basis of the formulas in Ref /8/. As a result the constancy condition of U-235 efficiency function is obtained in the core, having a special expression for the problem under consideration.

§5. Variational problem for maximum of breeding ratio (BR) in a breeder.

Consider a breeder with given demensions and given core and reflector materials. We assume the reactor fuel is U-238 and Pu-239 and there Pu-239 is accumulated. According to the determination the breeding ratio in a reactor throughout is:

$$BR = \frac{a_8}{a_{f9}} = \frac{\int\int_{V_0}\int_{\Omega} F(\vec{r},E,\vec{\Omega})\rho_8(\vec{r})\sigma_a^8(E)\,dE\,d\vec{\Omega}\,dV}{\int\int_{V_0}\int_{\Omega} F(\vec{r},E,\vec{\Omega})\rho_9(\vec{r})\sigma_{af}^9(E)\,dE\,d\vec{\Omega}\,dV} \qquad (8)$$

The relation between the U-238 concentration $\rho_8(\vec{r})$ and the Pu-239 concentration $\rho_9(\vec{r})$ in every reactor point $\vec{r}$ can be varied with the follo-

wing condition preserved:

$$\rho_8(\vec{r}) + \alpha \rho_9(\vec{r}) = \beta \qquad (9)$$

where α and β are some given fuel element parameters. What distribution of concentrations $\rho_8(\vec{r})$ and $\rho_9(\vec{r})$ in the reactor is required to achieve the maximum BR without K_{eff} variation?

Expressions for the variation of the basic functional BR and isoperimetric condition $\delta K_{eff} = 0$ have been taken from Ref/8/. Let $\rho_8(\vec{r})$ be a varying function because $\rho_9(\vec{r})$ is connected with it by the relation (9).

Taking into account consideration mentioned above we write:

$$\delta BR = \int_{V_p} \varphi(\vec{r}\ \rho_8)\, \delta\rho_8(\vec{r})\, dV \qquad (10)$$

$$\delta K_{eff} = \frac{1}{J_{fn}} \int_{V_p} \xi(\vec{r};\rho_8)\, \delta\rho_8(\vec{r})\, dV = 0 \qquad (11)$$

where functions $\varphi(\vec{r};\rho_8)$ and $\xi(\vec{r};\rho_8)$ are as follows:

$$\frac{\varphi(\vec{r};\rho_8)}{BR} = \iint_{\Delta\Omega}\left\{\left[\frac{\sigma_a^8(E)}{a_1} + \frac{\sigma_{af}^8(E)}{a\, a_{f9}}\right] - \left[\sigma_{a\,ifs}^8(E) - \frac{\sigma_{a\,ifs}^9(E)}{\alpha}\right]\Phi^+(\vec{r},E,\vec{\Omega})\right\} \times$$
$$\times F'(\vec{r},E,\vec{\Omega})\, dE\, d\vec{\Omega} + \int_{\Delta\Omega'}\int_{\Delta\Omega}\int_{\Delta E}\left\{\left[W^8(E'\to E,\vec{\Omega}\vec{\Omega}') - \frac{W^9(E'\to E,\vec{\Omega}\vec{\Omega}')}{\alpha}\right] +$$
$$+\left[\nu_f^8(E')\sigma_f^8(E')\chi_8(E,E',\vec{\Omega}\vec{\Omega}') - \frac{1}{\alpha}\nu_f^9(E')\sigma_f^9(E')\chi_9(E,E',\vec{\Omega}\vec{\Omega}')\right]\right\} \times$$
$$\times F'(\vec{r},E',\vec{\Omega}')\,\Phi^+(\vec{r},E,\vec{\Omega})\, dE'd\vec{\Omega}'dEd\vec{\Omega} \qquad (12)$$

$$\xi(\vec{r};\rho_8) = \int_{\Delta\Omega'}\int_{\Delta\Omega}\int_{\Delta E}\left\{\left[W^8(E'\to E,\vec{\Omega}\vec{\Omega}') - \frac{1}{\alpha}W^9(E\to E',\vec{\Omega}\vec{\Omega}')\right] +$$
$$+\left[\nu_f^8(E')\sigma_f^8(E')\chi_8(E,E',\vec{\Omega}\vec{\Omega}') - \frac{1}{\alpha}\nu_f^9(E')\chi_9(E,E',\vec{\Omega}\vec{\Omega}')\right]\right\} \times$$
$$\times F'(\vec{r},E',\vec{\Omega}')\, F^+(\vec{r},E,\vec{\Omega})\, dE'd\vec{\Omega}'dEd\vec{\Omega} -$$
$$-\int_{\Delta\Omega}\int_{\Delta E}\left[\sigma_{a\,ifs}^8(E) - \frac{1}{\alpha}\sigma_{a\,ifs}^9(E)\right]F'(\vec{r},E,\vec{\Omega})F^+(\vec{r},E,\vec{\Omega})\, dE\, d\vec{\Omega} \qquad (13)$$

By means of relations (10) and (11) we found Euler's equation for the problem under consideration:

$$\theta(\vec{r};\rho_8) = \frac{\varphi(\vec{r};\rho_8)}{\xi(\vec{r};\rho_8)} = const \qquad (14)$$

The condition obtained of constancy U-238 efficiency function in the reactor for problem under consideration can be used in numerical calculations to find the U-238 optimum distribution $\rho_8(\vec{r})$.

Chapter III. OPTIMIZING OF PHYSICAL CHARACTERISTICS OF RADIATION SHIELDING.

§1. The statement of problem.

The basic problem of design and calculation of reactor shielding is to find such distribution of shielding materials (heavy and light components, neutron absorber etc.), that satisfies some requirements. These requirements ussually consist in minimizing of a shielding weight or sizes under definite conditions (given dose on the shielding, heat generations etc.).

Thus, this problem may be formulated as a variational problem on optimum of some value F_0, for other values given, $F_1;F_2\ldots;F_k\ldots;F_m$.

In some simple cases this problem can be resolved as a classical variational one (E.Blizard, L.Kimel,E.Petrov,L.Pankratov,G.Lisochkin et al /20/,/21/,/22/). However, in more complex cases when a large change of radiation spectrum occurs (for instance, while introducing strong absorbers for capture gamma radiation depression), processes in shielding should be described by a complex multigroup system of equations and using of classical methods is difficult. Classical methods apply also to problems, when optimum is achieved in an internal region of permissible material concentration alterations ($0\leq\rho_q(\vec{r})\leq\rho_q^0$, where $\rho_q(\vec{r})$ is concentration of "q" material in point $\vec{r}$, ρ_q^0 is its maximum value). However, in practical problems there is a situation, when in some regions it is not advantageous to put some materials, and in other regions it is profitable to put a material with maximum density.

A general approach to the problem solution on optimum distribution of shielding materials may be based on using the method of consecutive approximations, step "n" of which is a transition from distribution $\rho_q^{(n)}(\vec{r})$ to $\rho_q^{(n+1)}(\vec{r})=\rho_q^{(n)}(\vec{r})+\delta\rho_q^{(n)}(\vec{r})$. Functions $\delta\rho_q^{(n)}(\vec{r})$ should be chosen in such a way that:

a)the minimized value F_0 would decrease as much as possible;

b)Values $F_1,F_2\ldots F_m$ would remain equal to given ones;

c)transition $\rho_q^{(n)}(\vec{r})\to\rho_q^{(n+1)}(\vec{r})$ would remain in region of permissible values of functions $\rho_q(\vec{r})$.

To choose $\delta\rho_q^{(n)}(\vec{r})$ it is necessary to know in what way the value F_k will be changed if the density of the materials changes in different points $\vec{r}$. If the value $\delta\rho_q^{(n)}(\vec{r})$ is rather small for the definition of variation δF_k formulas of small perturbation theory (Refs /7/,/10/) can be used, where F_k is expressed by means of functions of flux and "danger" of neutrons, which are solutions of the basic equation and the adjoint equation in the preceding iteration. These formulas permit to calculate functions:

$$h_q^{\kappa}(\vec{r}) = -\frac{\delta F_\kappa}{\delta\rho_q(\vec{r})}$$

which are material "q" efficiency in the point $\vec{r}$ relative to the value F_k. The knowledge of efficiency functions permits reasonable choice of value $\delta\rho_q^{(n)}(\vec{r})$ in every iteration. For example, in the absolute extremum problem $\delta\rho_q^{(n)}(\vec{r})$ can be chosen in form $\delta\rho_q^{(n)}(\vec{r})=\Delta h_q^{(n)}(\vec{r})$ in such a way that if q^{th} material efficiency relative to F_0 is maximum, the greater amount of this material is added, where $h_q(\vec{r})$ is negative its amount is decreased. Of course, in order to use the small perturbation theory it is necessary to choose rather small value of $\delta\rho_q(\vec{r})$, this can be ensured

by choosing a proper value A.

For the construction of numerical algorithm of optimum solution investigation it can be used the linear programming ideas.

§2. The perturbation theory application to shielding characteristics. Shielding material efficiency functions.

In general physical characteristics of a shielding $F_0, F_1 \ldots F_k \ldots F_m$ depend on neutron flux, gamma quantums and distribution of shielding materials $\rho_q(\vec{r})$.

The neutron flux distribution is described for the steady state by the equation:

$$\vec{\Omega}\nabla\varphi + \Sigma(E)\varphi - \int d\vec{\Omega}' \int dE' \varphi(\vec{r},\vec{\Omega}',E')\,\Sigma(\vec{\Omega}'\to\vec{\Omega},E'\to E) = q(\vec{r},\vec{\Omega},E) \quad (1)$$

where $\Sigma(E)$ is a total macroscopic cross-section of interaction; $\Sigma(\vec{\Omega}'\to\vec{\Omega},\ E'\to E)$ is the differential section defining the transition of neutrons from the beam $(\vec{\Omega}',E')$ to the beam $(\vec{\Omega},E)$ (here it is considered an elestic and inelastic transitions and fission too); $q(\vec{r},\vec{\Omega},E)$ is the distribution of neutron sources. A space for which we find a solution of equation (1) occupies the volume V, limited by surfaces S.

The different physical characteristics of a shielding can be written in the form:

$$F_k = \int_V \varphi(\vec{r},\vec{\Omega},E) P_k(\vec{r},\vec{\Omega},E)\, d\vec{\Omega}\, dE\, d\vec{r} + \Phi[\rho_q(\vec{r})] \quad (2)$$

where the first term defines values due to the different neutron interaction processes while passing surface, a heat generation from neutrons and capture gamma-quantums, a neutron dose, etc. The term $\Phi[\rho_q(\vec{r})]$ defines values due to external gamma-quantum sources (for instance, gamma-quantums form active core). Here we have used the approximate method to consider gamma-quantum, although its description by a kinetic equation should not be a principal difficulty.

The shielding weight is a functional in form:

$$F_0 = \int \Sigma_q \rho_q(\vec{r})\, dV$$

Consider an adjoint with (1) equation:

$$-\vec{\Omega}\nabla\varphi_k^+ + \Sigma(E)\varphi_k^+ - \int d\vec{\Omega}' \int dE' \varphi_k^+(\vec{r},\vec{\Omega}',E')\,\Sigma(\vec{\Omega}\to\vec{\Omega}',E\to E') = \frac{\delta F_k}{\delta\varphi} \quad (3)$$

where $\delta F_k/\delta\varphi$ is a functional derivative of F_k with respect to φ.

In case of linear dependence from φ for functionals of type (2):

$$\frac{\delta F_k}{\delta\varphi} = P_k(\vec{r},\vec{\Omega},E)$$

As has been shown in Refs /6/,/7/,/10/,/13/, the adjoint function $\varphi_k^+(\vec{r},\vec{\Omega},E)$ describes a variation of value F_k, if one additional neutron has been injected in point $(\vec{r},\vec{\Omega},E)$. In shielding this function defines a neutron "danger" relative to the neutron dose over a shielding, to gamma-radiation dose, to a heat generation, etc.

A knowledge of flux and "danger" functions permits to formulate a perturbation theory for different functionals.

It can be shown, that

$$\delta F_k = -\int \varphi_k^+(\vec{r},\vec{\Omega},E)\,\delta L\,\varphi(\vec{r},\vec{\Omega},E)\, d\vec{r}\, d\vec{\Omega}\, dE + \int \delta P(\vec{r},\vec{\Omega},E)\,\varphi(\vec{r},\vec{\Omega},E)\, d\vec{r}\, d\vec{\Omega}\, dE +$$
$$+ \int \delta q(\vec{r},\vec{\Omega},E)\,\varphi^+(\vec{r},\vec{\Omega},E)\, d\vec{r}\, d\vec{\Omega}\, dE + \int \Sigma_q \frac{\delta\Phi}{\delta\rho_q}\,\delta\rho_q(\vec{r})\, d\vec{r} \quad (4)$$

where δL, δp and δq are variations, due to a material density variation, $\delta\rho_q(\vec{r})$, at the kinetic equation operator (1), and the right parts of equations (1) and (3) too.

As it has been shown in §1 the efficiency material function "q" relative to different functionals F_k has form:

$$h_q^k(\vec{r}) = -\frac{\delta F_k}{\delta \rho_q(\vec{r})} \qquad (5)$$

The efficiency function defines the functional variation F_k while inserting a unit quantity of the material "q" in point $\vec{r}$. Values $h_q^k(\vec{r})$ can be calculated using the perturbation theory formulas (4).

Then a change in functionals δF_k at varying material q density in a unit volume near a point $\vec{r}$ by a value $\delta\rho_q(\vec{r})$ will be equal to:

$$\delta F_k = h_q^k(\vec{r})\,\delta\rho_q(\vec{r}) + \int \frac{\delta\Phi}{\delta\rho_q}\,\delta\rho_q(\vec{r})$$

Thus having information about efficiency functions know what variations in different functionals will result from some small variation of the material densities $\delta\rho_q(\vec{r})$.

§3. The minimizing procedure.

With consecutive operations for minimizing some functional F_0, provided other functionals $F_1 \ldots F_m$ are under definite conditions:

$$F_k \le D_k, \quad k = 1, 2, \ldots m \qquad (6)$$

it is necessary to find such values $\delta\rho_q(\vec{r})$ in each stage that should lower the value of the minimized functional under the condition (6):

Let us write $\rho_q(\vec{r})$ in the form

$$\rho_q(\vec{r}) = \rho_q^0 U_q(r)$$

where ρ_q^0 is a theoretical density of material q,

and $0 \le U_q(r) \le 1$ and $\sum_q U_q(r) \le 1$ (7)

$U_q(r)$ called further the control, will be considered as piece-constant and equal to U_{qj} at interval Δr_j. Let U_{qj} be in vector form $\vec{U}_j$, which has components U_{qj}.

Let $\delta\rho_{qj}$ be in form $\delta\rho_{qj} = S_{qj}\,\delta U_{qj}$, where δU_{qj} are the variations of vector $\vec{U}_j$, the using of which with positive weight does not take out $\vec{U}_j$ from the region (7) δU_{qj} and $-\delta U_{qj}$ are considered to be different, some of them can be "barred". S_{qj} are some numbers, satisfied by limits:

$$0 \le S_{qj} \le S_{qj}^0 \qquad (8)$$

Vectors $\vec{h}_{qj}$ correspond to variations δU_{qj}, vectors components are defined above by efficiencies h_{qj}^k of material q in a point j relative to the functional F_k.

Vector $\vec{h}_{qj}$ appertains to a space R_{m+1}, measurement of which is $(m+1)$. In the construction of vectors $\vec{h}_{qj}$ in the space R_{m+1} it is necessary to introduce a metric, as different components of vectors $\vec{h}_{qj}$ have different measurements and different orders of values. The metric can be introduced for example, by means of variation of scales on the different components (i.e. by action of some diagonal matrix) so that average values of vector components $\vec{h}_{qj}$ should become approximately equal.

All further constructions are based on the linear small perturbation theory. Now the variation problem is in the following form.

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To find such numbers S_{qj} satisfying to limitations (8), that for the control

$$U^{*}_{qj} = U_{qj} + S_{qj}\,\delta U_{qj}$$

conditions (6) and (7) should be fulfilled and with this an increment of the minimizing funotional should be negative

$$\delta F_0 = \sum_{qj} S_{qj}\, h^{0}_{qj}\, \Delta\tau_j$$

A number of points

$$\vec{\xi} = \sum_{qj} S_{qj}\, \vec{h}_{qj}\, \Delta\tau_j$$

at all S_{qj}, subordinated by the inequalities (8), have made a convex closed polyhedron R, which should be called an attainable region. R is a limited region of the spaoe R_{m+1}.

To definition let us consider a oase, when conditions (6) have a form of equalities. Then the problem is reduced to a search of the point of the intersection of the ray l_0 with the direction (-1,0,0...0) with a boundary of region R (in the common case to an intersection point of some spaoe, the points of whioh satisfy to conditions (6) with the boundary of the region.

Let us consider one of simple but in practice sufficiently effective methods to define S_{qj}. (In detail this problem has been considered in Ref. /23/).

The variation procedure consists of series of the elementary actions, in everyone of which the control varies only on one $\Delta\tau_j$ for one component q. A variation is chosen in such a way that as a result the minimized functional being more lowered at a least possible disturbance of the conditions (6). Suoh variation is chosen for the conditions of minimum angle between the vectors $\vec{h}_{qj}$ and $\vec{l}_0$. Let pass from U_{qj} to $U^{*}_{qj} = U_{qj} + S_{qj}\cdot\delta U_{qj}$. As consequence of this there will be a displacement of the point $\vec{\xi}$ to the point $\vec{\xi}^{*} = \vec{\xi} + S_{qj}\vec{h}_{qj}\Delta\tau_j$, distanoe of which from axis l_0 is $\rho(\vec{\xi}^{*})$.

The seoond elementary variation act is the restoration of boundary oonditions(6). In this case the control variation is chosen from condition of minimum angle between vectors $\vec{h}_{qj}$ and $(0,\xi^1,\xi^2\ldots\xi^k\ldots\xi^m)$.

All vectors $\vec{h}_{qj}$ available at our disposal are considered identically. In that the iteration is oompleted, vectors $\vec{h}_{qj}$ are counted again at controls $\vec{U}^{*}_{qj}$ obtained in the previous iteration and then a new iteration is begun.

Other method of values S_{qj} determination is possible. It consists in the choice of some distance ρ_1 from the axis (-1,0,0...0)(it is some boundary condition non-conservance), whioh tends to zero in the iteration procedure. Then $\vec{S}_{qj}$ is determined from relation:

$$\rho(\vec{\xi}) = \rho(S_{qj}, \vec{h}_{qj}, \Delta\tau_j) = S_{qj}\,\rho(\vec{h}_{qj})\cdot\Delta\tau_j = \rho_1$$

The process is stopped, if it is not possible to select such S_{qj} by means of whioh the functional F_0 can be reduced with conservance of condition (6).

The optimality criterion can be formulated in such a way. The optimal composition is one (such values of control U_{qj}), vectors $\vec{h}_{qj}$ of which form a convex cone, not containing the minimized direction vector $\vec{l}_0$.

Denoted minimization prinoiple is applied, of course, to reactor problems too.

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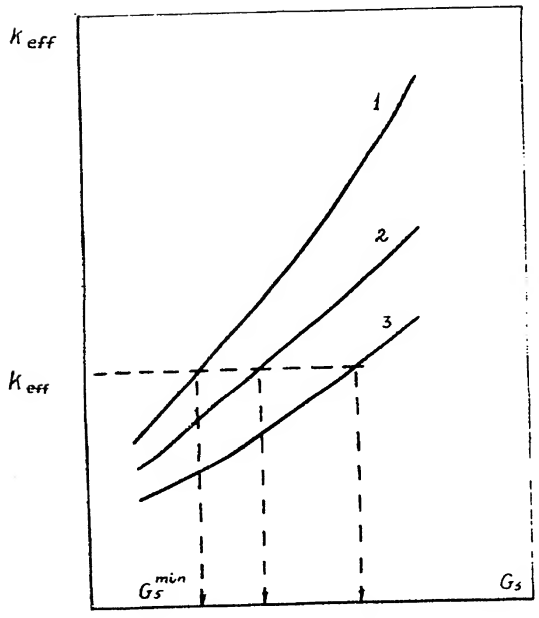

Fig.1. SCHEMATIC DIAGRAM OF DEPENDENCES $K_{eff}=F(G_5)$. curve (1) - $\rho_5(r)$ provides K_{eff}^{max}; curve (2) - $\rho_5=$ const; curve (3) - $\rho_5=\rho_5(r)$, but not optimal; K'_{eff} is required value of K_{eff}.

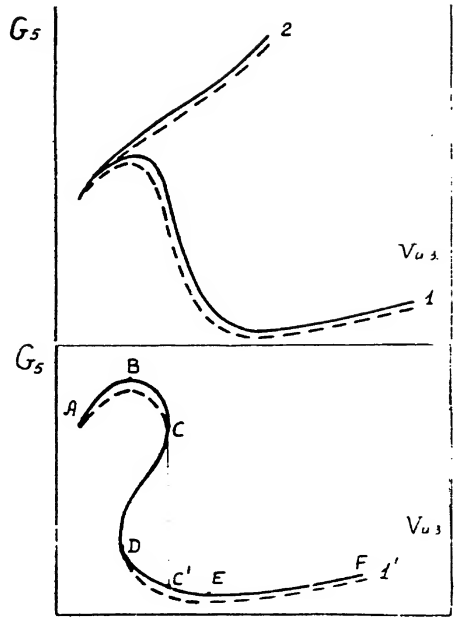

Fig.2. SCHEMATIC DIAGRAM OF DEPENDENCE $G_5=f(V_{u.})$. ——critical mass at $\rho_5=$const; --- minimized critical charge; 1.1 U-235 is diluted by elastic scattering moderator; 2. U-235 is diluted by inelastic scatterer.

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